EE224-

Q1) A continuous time periodic signal x(t) is real valued and has a fundamental period T=8. The nonzero Fourier series coefficients for x(t) are specified as

$$a_1 = a_{-1}^* = j, \qquad a_5 = a_{-5} = 2.$$

Express x(t) in the form $x(t) = \sum A_k \cos(\omega_k t + \phi_k)$.

Q2) Let x(t) be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$$

and let

$$h(t) = u(t) - u(t-2).$$

(a) Is x(t) periodic?
(b) Is x(t)*h(t) periodic?

SOLUTIONS

S1) Using Fourier series synthesis

$$x(t) = a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_5 e^{j5(2\pi/T)t} + a_{-5} e^{-j5(2\pi/T)t}$$ **(7 points)**

$$x(t) = j e^{j(2\pi/8)t} - j e^{-j(2\pi/8)t} + 2 e^{j5(2\pi/8)t} + 2 e^{-j5(2\pi/8)t}$$ **(6 points)**

$$x(t) = -2\sin\left(\frac{\pi t}{4}\right) + 4\cos\left(\frac{5\pi t}{4}\right)$$ **(6 points)**

$$x(t) = -2\cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) + 4\cos\left(\frac{5\pi t}{4}\right)$$ **(6 points)**

S2) Taking the inverse Fourier transform of X(jω), we obtain

$$x(t) = \frac{1}{2\pi} + \frac{1}{2\pi} e^{j\pi t} + \frac{1}{2\pi} e^{j5\pi t}$$ **(5 points)**

(a) The signal x(t) is a constant summed with two complex exponentials whose fundamental frequency are 2π/5 rad/sec and 2 rad/sec. These two complex exponentials are not harmonically related. Therefore, the signal is **<u>NOT PERIODIC</u>**. **(4 points)**
(b) Consider the signal y(t)=x(t)*h(t). From the convolution property, we know that $Y(j\omega) = X(j\omega).H(j\omega)$. Also,

$$H(j\omega) = e^{-j\omega} \frac{2\sin\omega}{\omega}$$ **(4 points)**

The function H(jω) is zero when ω=kπ, where k is a nonzero integer. Therefore,

$$Y(j\omega) = X(j\omega).H(j\omega) = \delta(\omega) + \delta(\omega - 5)$$ **(4 points)**

This gives

$$y(t) = \frac{1}{2\pi} + \frac{1}{2\pi} e^{j5\pi t}$$ **(4 points)**

Y(t) is a complex exponential summed with a constant. We know that a complex exponential is periodic. Adding a constant to a complex exponential does not affect its periodicity. Y(t) will be a signal with a fundamental frequency of 2π/5 rad/sec. **(4 points)**