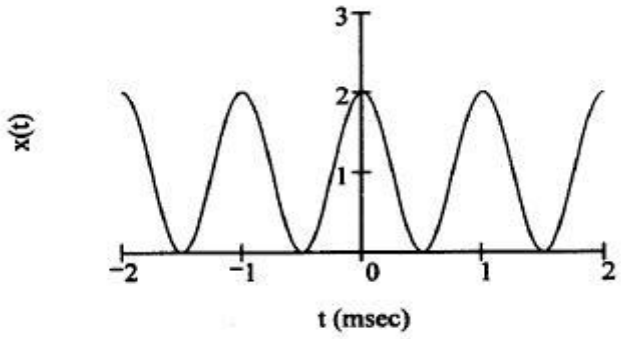
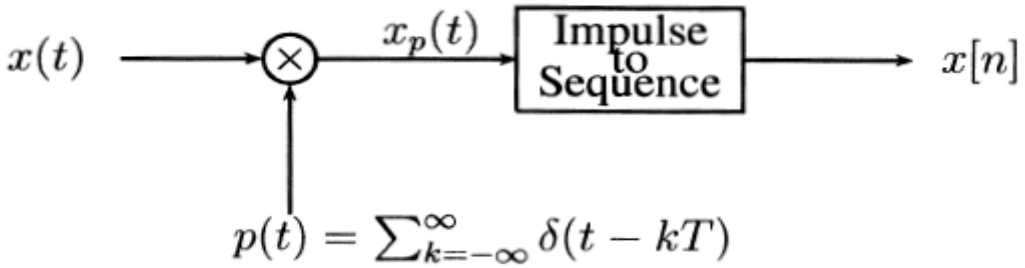
06.06.2007

EE 224 Semester Exam- Solution

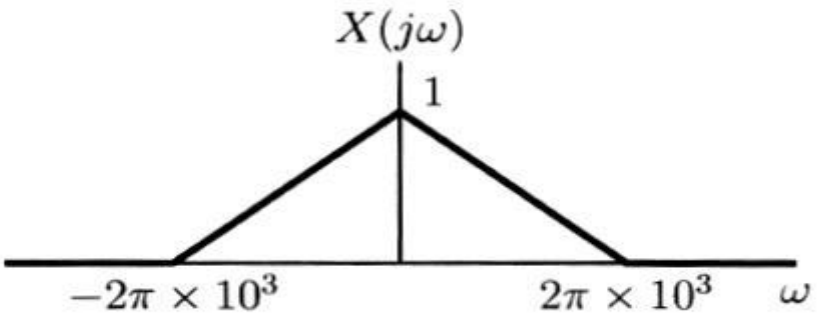
Q1) (20pts) Express the sinusoidal signal shown to below as a Fourier series in complex exponential form.



Q2) (30pts) Consider the following system.



The Fourier transform of x(t), $x_p(t)$ and x[n] are denoted respectively X(jω), $X_p(j\omega)$ and $X(e^{j\Omega})$. If X(jω) is as shown below and $T=0{,}5.10^{-3}$ sec, provide labeled sketches of $X_p(j\omega)$ and $X(e^{j\Omega})$.



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S1) The signal is a cosinus plus a dc offset, so there are only 3 terms in the series. We have $T=10^{-3}$ sec.

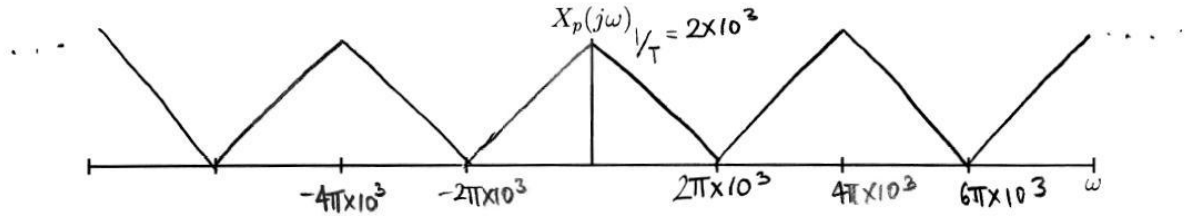
$$x(t) = 1 + \cos\left(\frac{2\pi t}{T}\right) = 1 + \frac{1}{2}e^{\frac{j2\pi t}{T}} + \frac{1}{2}e^{\frac{-j2\pi t}{T}}$$

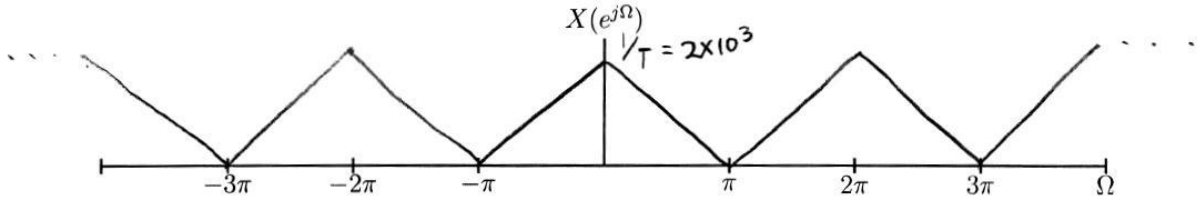
This makes $X_{-1}=1/2$, $X_1=1/2$ and $X_0=1$(dc offset) (10pts)

And the series becomes

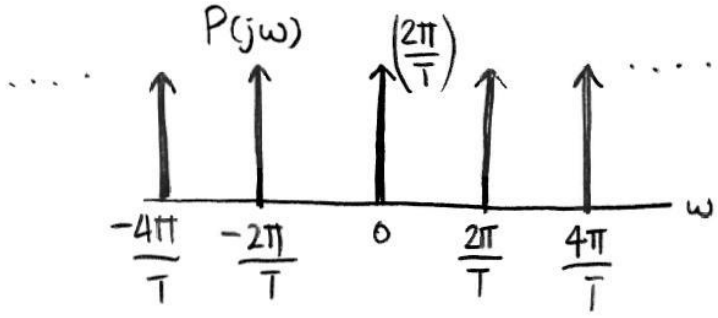
$$x(t) = 1 + \frac{1}{2}e^{\frac{j2\pi t}{T}} + \frac{1}{2}e^{\frac{-j2\pi t}{T}} = 1 + \frac{1}{2}e^{-j2\pi 1000t} + \frac{1}{2}e^{j2\pi 1000t} \quad (10\text{ pts})$$

S2) (10pts)





(10pts)



(5pts)

$$X_p(j\omega) = \frac{1}{2\pi}P(j\omega) * X(j\omega) \text{ and } \frac{2\pi}{T} = \frac{2\pi}{0{,}5.10^{-3}} = 4\pi.10^{-3} \text{ (5pts).}$$

$X(e^{j\Omega})$ is just $X_p(j\omega)$ with a scaling on the frequency axis by $T=0{,}5\ldots10^{-3}$.

Q3) (30pts) Let the Fourier transform of discrete time signal is given as $X(\omega) = 1 + 2e^{-j\omega} + 2e^{-j2\omega}$

a) Calculate and plot the magnitude and phase of X(ω).

b) By inspecting the figures, is the time domain sequence periodic? Why?

c) Find the time domain sequence using the following table and properties of the discrete time Fourier transform.

Time Domain x[n]	Frequency Domain X(ω)
$\delta[n]$	1
$u[n]$	$\frac{1}{1-e^{-j\omega}}$
e^{jn}	$2\pi\,\delta(\omega)$

Q4) (20pts) Assume that you have a discrete data set collected from real time environment.

a) What would you do to in order to get information about underlying periodicities?

b) What would you do if the number of data is not power of 2?

c) Why do we want that the number of data is power of 2?

d) How can you improve the resolution to get more information?

e) If there exist a large peak at zero frequency, what does this mean? How can it be removed?

f) What is the meaning of the peaks in the obtained spectrum?

g) Assume that the data is taken with a sample period of 1 minute and the number of data is 1024, the peak resides in indice m=240. Calculate the frequency and period associated with that peak.