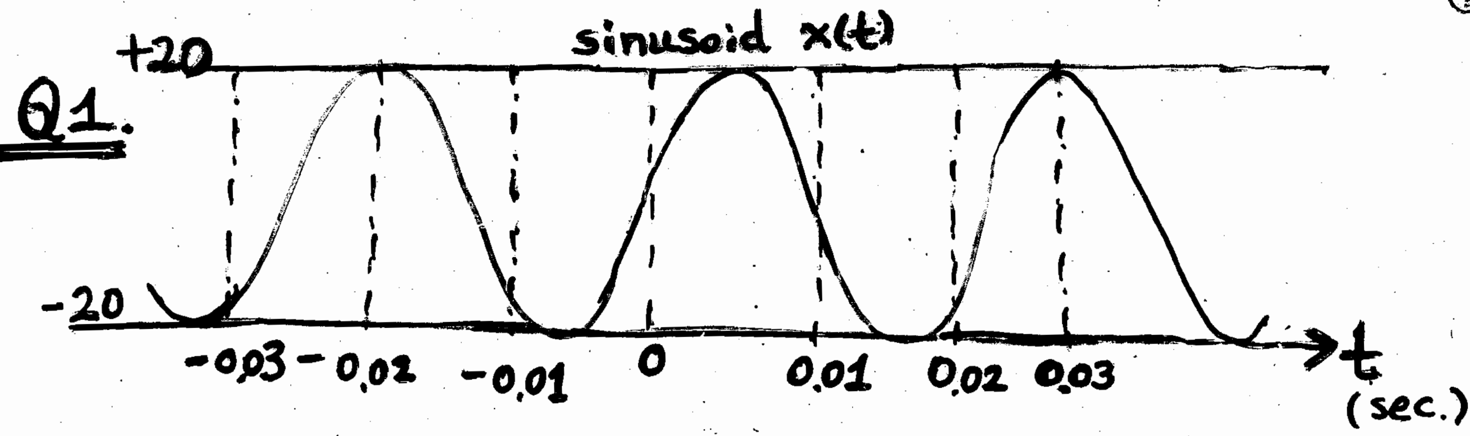
Q1.



What is the mathematical expression of the above sinusoid?

$$x(t) = A \cdot \cos(\omega_0 t + \phi)$$

$A = ?$ $\quad \omega_0 = ?$ $\quad \phi = ?$

Let's denote period as T $\Rightarrow$ $2T = 0.03 - (-0.02)$
$= 0.05$ sec.

$\Rightarrow T = 0.025$ sec.

$$\Rightarrow f_0 = \frac{1}{T} = \frac{1}{0.025} = 40\,Hz.$$

$\Rightarrow \omega_0 = 2\pi(40)$ rad/sec.

$A = 20$

$x(t) = 20 \cos[2\pi(40)t + \phi]$

$$20 = 20 \cos[2\pi(40)(0.03) + \phi]$$

$$\Rightarrow \cos(2.4\pi + \phi) = 1$$

$$\Rightarrow 2.4\pi + \phi = 2\pi \Rightarrow \phi = -0.4\pi$$

$$\boxed{x(t) = 20 \cos[2\pi(40)t - 0.4\pi]}$$

Q2. Define $x(t) = 3\cos\left(\omega_0 t - \frac{\pi}{4}\right)$

For $\omega_0 = \frac{\pi}{5}$, make a plot of $x(t)$ over the range

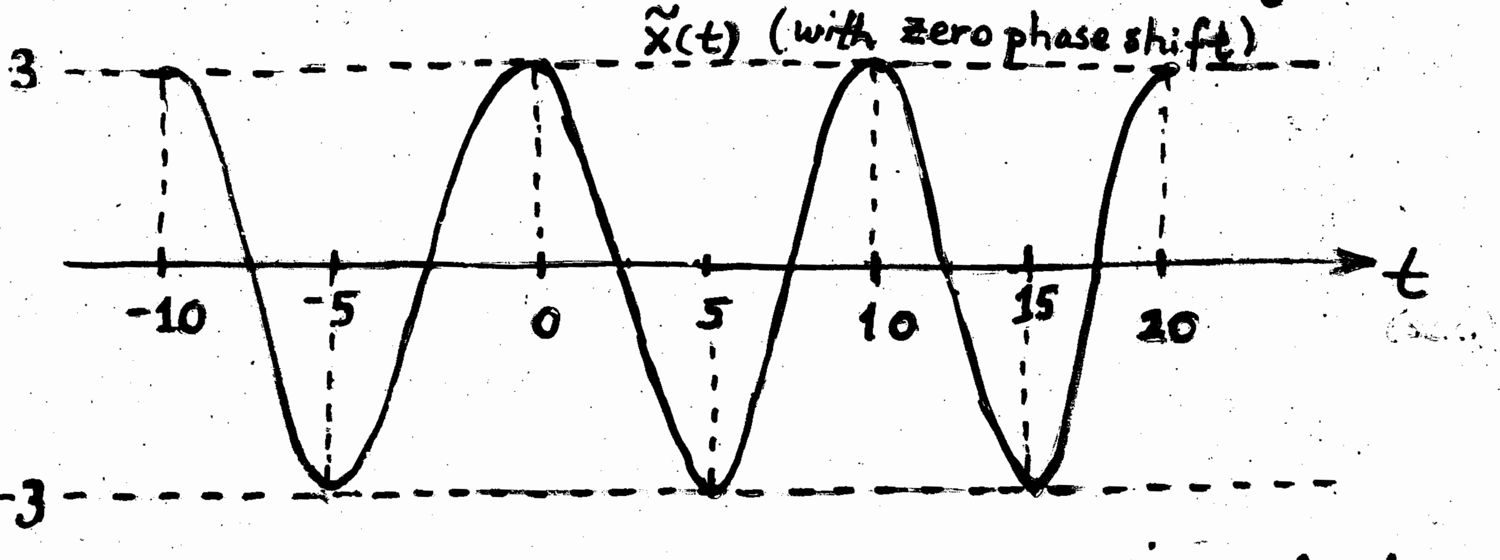
$-10_{(sec)} \le t \le 20_{(sec)}$.

$$\omega_0 = \frac{\pi}{5} \Rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{\pi/5}{2\pi} = \frac{1}{10} \text{ Hz.}$$

$$\Rightarrow \text{period } T = \frac{1}{f_0} = \frac{1}{\frac{1}{10}} = 10 \text{ sec.}$$

$$x(t) = 3\cos\left(\frac{\pi}{5}t - \frac{\pi}{4}\right)$$

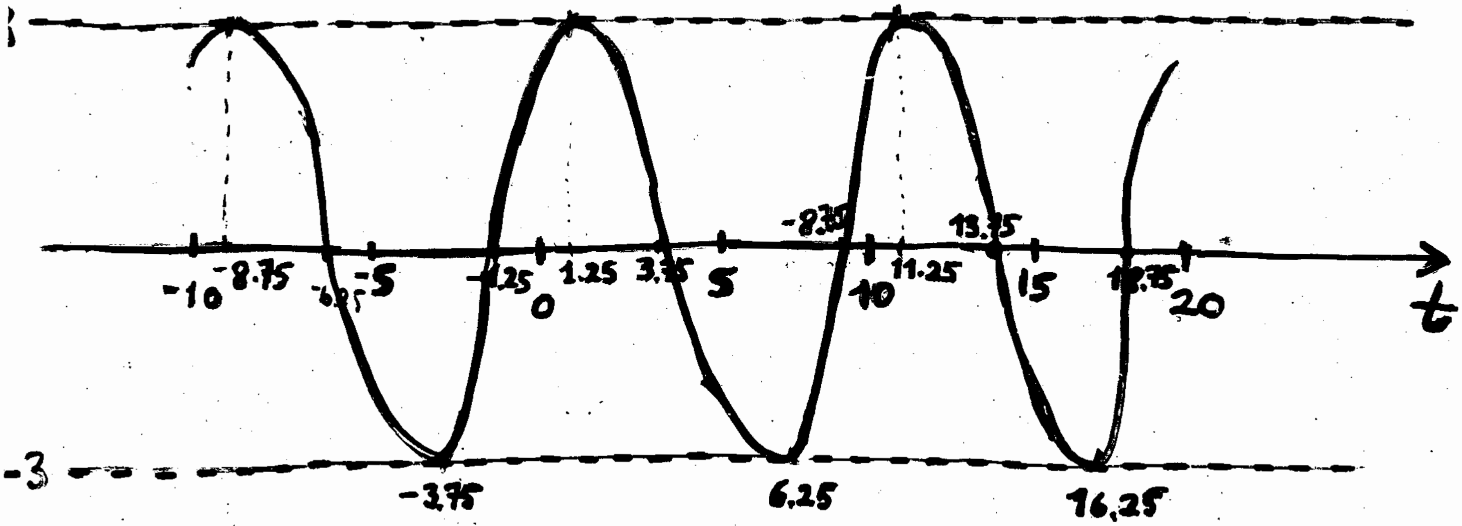
let's first plot $\tilde{x}(t) = 3\cos\left(\frac{\pi}{5}t\right)$



remember the relation between time shift t_1 and phase shift ϕ

$$\boxed{t_1 = -\frac{\phi}{\omega_0}} = -\frac{-\frac{\pi}{4}}{\frac{\pi}{5}} = \frac{5}{4} = 1.25$$

$$\Rightarrow x(t) = \tilde{x}(t - t_1) = \tilde{x}(t - 1.25)$$



check the answer: $t = 3.75$

* $x(3.75) = 3\cos\left(\frac{\pi}{5}3.75 - \frac{\pi}{4}\right) = 3\cos(0.75\pi - 0.25\pi)$
$= 3\cos\left(\frac{\pi}{2}\right) = 0$ ✓

* $x(16.25) = 3\cos\left(\frac{\pi}{5}(16.25) - \frac{\pi}{4}\right) = 3\cos(3.25\pi - 0.25\pi)$
$= 3\cos(3\pi) = -3$ ✓

* $x(11.25) = 3\cos\left(\frac{\pi}{5}(11.25) - \frac{\pi}{4}\right) = 3\cos(2.25\pi - 0.25\pi)$
$= 3\cos(2\pi) = 3$ ✓

Q3. Solve the following equation for θ.

$$\mathrm{Re}\{(1+j)e^{j\theta}\} = -1$$

$$\mathrm{Re}\{(1+j)(\cos\theta + j\sin\theta)\} = -1$$

$$\mathrm{Re}\{\cos\theta + j\sin\theta + j\cos\theta - \sin\theta\} = -1$$

$$\cos\theta - \sin\theta = -1$$

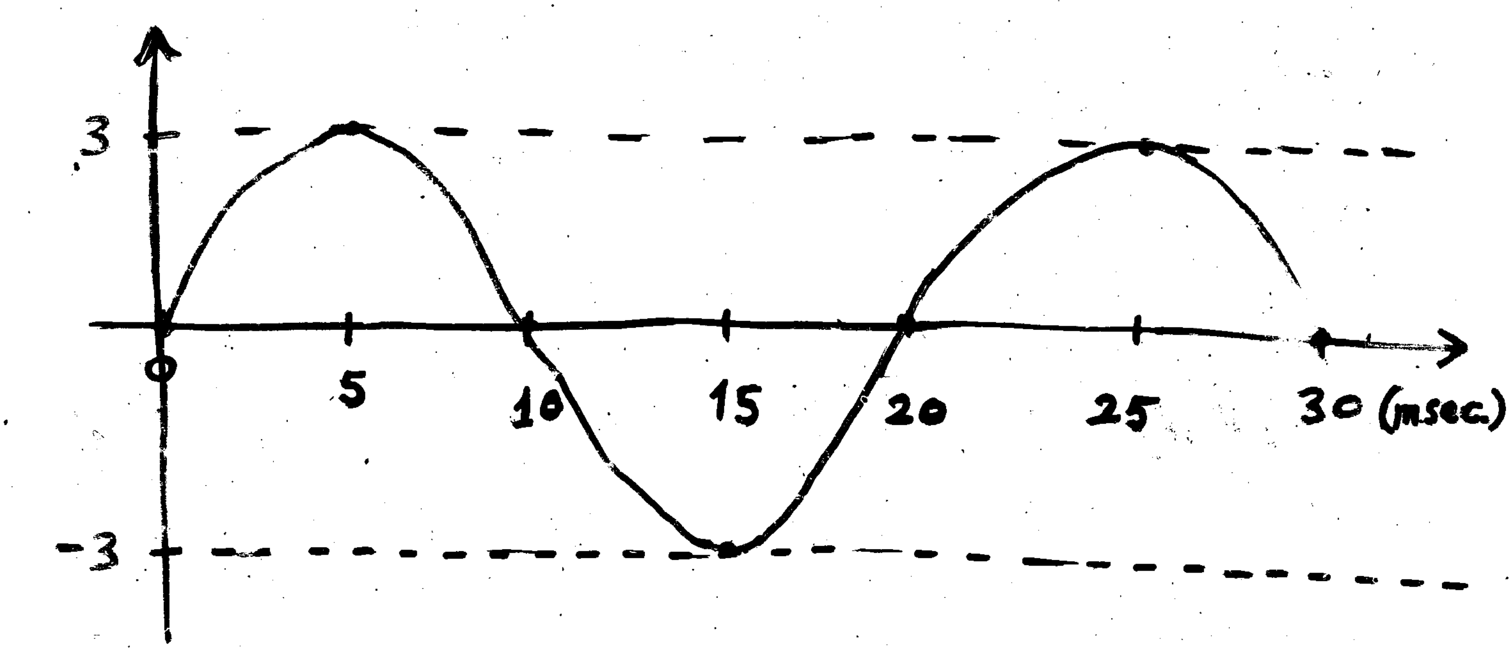
$\Rightarrow 1 + \cos\theta = \sin\theta \longrightarrow \theta = \frac{\pi}{2}$

$1 + \cos\frac{\pi}{2} \overset{?}{=} \sin\frac{\pi}{2}$

Slaytlar dersten sonra dağıtılsın

Question : Consider the following analog sinusoidal signal ; $x_a(t) = 3\sin(100\pi t)$

a) Sketch the signal $x_a(t)$ for $0 \le t \le 30$ msec.



b) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/sec. Determine the frequency of the discrete-time signal $x[n] = x_a(nT_s)$, $T_s = \frac{1}{F_s}$ and show that $x[n]$ is periodic.

$$x[n] = x_a\left(n \cdot \frac{1}{300}\right) = 3\sin\left[100\pi\left(\frac{n}{300}\right)\right] = 3\sin\left[\frac{\pi}{3} \cdot n\right]$$

$$= 3\sin\left[2\pi \cdot \frac{1}{6} \cdot n\right]$$

$\Rightarrow$ discrete frequency $\frac{1}{6}$

$\Rightarrow$ period : $N=6$ samples.

$x[n+N] = x[n+6] = 3\sin\left[\frac{\pi}{3}(n+6)\right] = 3\sin\left(\frac{\pi n}{3} + 2\pi\right) = 3\sin\left[\frac{\pi n}{3}\right]$

$$x[n] = 3\sin\left[\frac{\pi n}{3}\right]$$

c) Compute the sample values in one period of $x[n]$.
What is the period of the discrete-time signal in miliseconds?

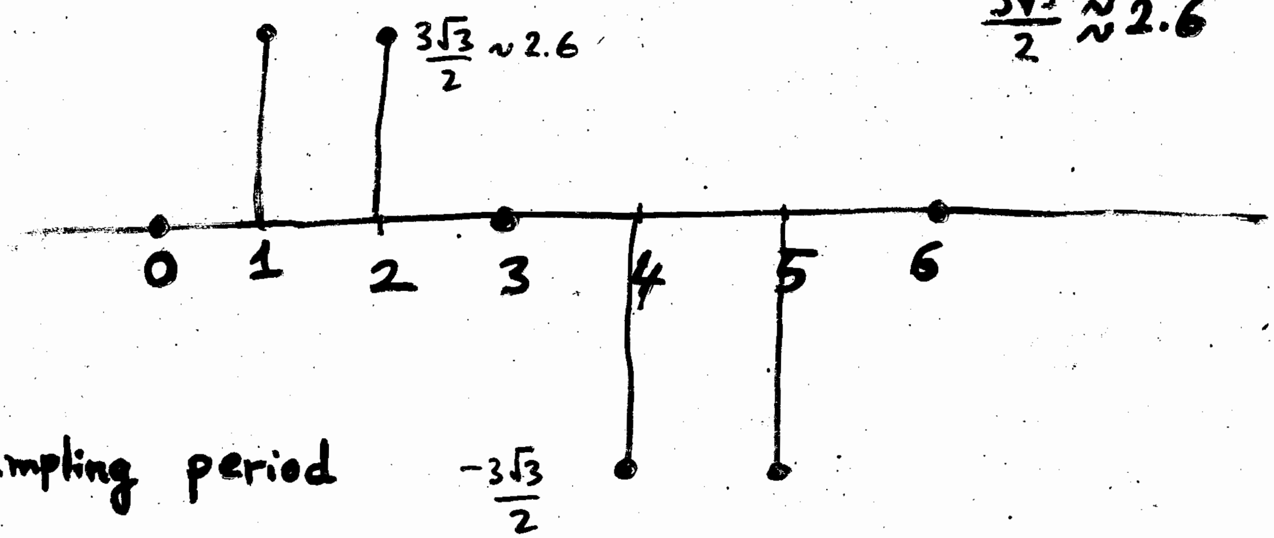
$x[0] = 3\sin[0] = 0$ $\qquad$ $x[1] = 3\sin\left[\frac{\pi}{3}\right] = \frac{3\sqrt{3}}{2}$

$x[2] = 3\sin\left[\frac{2\pi}{3}\right] = \frac{3\sqrt{3}}{2}$ $\qquad$ $x[3] = 3\sin[\pi] = 0$

$x[4] = 3\sin\left[\frac{4\pi}{3}\right] = -\frac{3\sqrt{3}}{2}$ $\qquad$ $x[5] = 3\sin\left[\frac{5\pi}{3}\right] = -\frac{3\sqrt{3}}{2}$

$x[6] = 3\sin[2\pi] = 0$

$\frac{3\sqrt{3}}{2} \approx 2.6$



sampling period

$T_s = \frac{1}{300}$ $\Rightarrow$ period of $x[n]$ in miliseconds:

$$6 \cdot \frac{1}{300} = 0{,}02 \text{ sec} = 20 \text{ msec}.$$

d) Can you find a sampling rate F_s such that the signal $x[n]$ reaches its peak value of 3.

Answer 200 samples

<u>Question:</u> A digital communication link carries binary-coded words representing samples of an input signal: $x_a(t) = 3\cos(600\pi t) + 2\cos(1800\pi t)$

The link is operated at 10,000 bits/sec. and each input sample is quantized into 1024 different voltage levels.

a) What is the sampling frequency?

1024 different voltage levels $\Rightarrow$ $2^{10} = 1024$

$\rightarrow$ each sample is represented by 10 bits

$\Rightarrow$ in one second $\dfrac{10{,}000 \text{ bits}}{10 \text{ bits}} = 1000$ samples/sec. $= F_s$

b) What is the Nyquist rate for the signal $x_a(t)$?

$F_{Nyquist} = 1800$ Hz.

c) What is the period of the resulting discrete-time signal?

$$x[n] = x_a\left(n\frac{1}{F_s}\right) = x_a\left(n\frac{1}{1000}\right) = 3\cos\left[600\pi \frac{n}{1000}\right] + 2\cos\left[1800\pi \frac{n}{1000}\right]$$

$$= 3\cos\left[2\pi \cdot \frac{3n}{10}\right] + 2\cos\left[2\pi \cdot \frac{9n}{10}\right]$$

$\Rightarrow N = 10.$

Soru: biri 4, diğeri 5 ??? olursa ???

d) What is the resolution Δ (quantizer step size)?

Q-4. Simplify following complex identities:

i) $3e^{j\frac{\pi}{3}} + 4e^{-j\frac{\pi}{6}} = 3\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right) + 4\left(\cos\frac{\pi}{6} - j\sin\frac{\pi}{6}\right)$

$= 3\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$

$= \frac{3}{2} + j\frac{3\sqrt{3}}{2} + 2\sqrt{3} - j2 = \left(2\sqrt{3} + \frac{3}{2}\right) + j\left(\frac{3\sqrt{3}}{2} - 2\right)$

ii) $Re\left\{j e^{-j\frac{\pi}{3}}\right\} = Re\left\{j\left(\cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right)\right\}$

$= Re\left\{j\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right\} = Re\left\{j\frac{1}{2} + \frac{\sqrt{3}}{2}\right\} = \frac{\sqrt{3}}{2}$

iii) $(\sqrt{3} - j3)^{1/3} = ?$

$|\sqrt{3} - j3| = \sqrt{3 + 9} = 2\sqrt{3}$

$\theta = \arctan\frac{-3}{\sqrt{3}} = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$\Rightarrow \sqrt{3} - j3 = 2\sqrt{3}\, e^{-j\frac{\pi}{3}}$

$\Rightarrow (\sqrt{3} - j3)^{1/3} = \left(2\sqrt{3}\, e^{-j\frac{\pi}{3}}\right)^{1/3} = (2\sqrt{3})^{1/3} \cdot e^{-j\frac{\pi}{9}}$

$= 1.513\, e^{-j\frac{\pi}{9}} = 1.513\cos\left(\frac{\pi}{9}\right) - j1.513\sin\left(\frac{\pi}{9}\right.$

Q5 phase of a sinusoid can be related to time-shifts as follows:

$$x(t) = A\cos(2\pi f_0 t + \underset{\text{Phase}}{\phi}) = A\cos[2\pi f_0 (t - \underset{\text{time shift}}{t_1})]$$

$$\Rightarrow \boxed{\phi = -2\pi f_0 t_1} \Rightarrow \boxed{\phi = -\omega_0 t_1} \Rightarrow \boxed{t_1 = \frac{-\phi}{\omega_0}}$$

(Assume period of the wave $T_0 = 8$ sec.)

TRUE OR FALSE ?

a) "when $t_1 = -2$ sec, the value of the phase is $\phi = \frac{\pi}{2}$."

b) "When $t_1 = 3$sec, the value of the phase $\phi = \frac{3\pi}{4}$."

c) "When $t_1 = 7$sec, the value of the phase is $\phi = \frac{\pi}{4}$"

a) $\phi = -2\pi f_0 t_1 = -2\pi \frac{1}{T_0} t_1 = -2\pi \frac{1}{8}(-2) = \frac{\pi}{2}$?

b) $\phi = -2\pi f_0 t_1 = -2\pi \frac{1}{T_0} t_1 = -2\pi . \frac{1}{8} 3 = -\frac{3\pi}{4}$?

c) $\phi = -2\pi f_0 t_1 = -2\pi \frac{1}{T_0} . t_1 = -2\pi . \frac{1}{8} . 7 = \frac{-7\pi}{4}$?

Q.6. Consider the analog signal

$$x(t) = 3\cos(100\pi t)$$

a) Determine the minimum sampling rate (Nyquist rate) to recover the original signal from its samples.

b) Suppose that signal is sampled at $F_s = 200$ Hz. What is the discrete time signal obtained after sampling.

c) Suppose that signal is sampled at $F_s = 75$ Hz. What is the discrete time signal obtained after sampling.

d) What is the frequency of a sinusoid that gives identical samples to those obtained in part c).

a) Nyquist rate is <u>twice</u> the max. frequency of $x(t)$.

$f_0 = 50$ Hz. $\Rightarrow$ Nyquist rate $= 100$ Hz.

b) $F_s = 200$ Hz. $\Rightarrow$ $T_s = \frac{1}{F_s} = \frac{1}{200} = 0.005$

F_s → sampling freq.

T_s → sampling interval

$x[n] = x(nT_s) = 3\cos[100\pi n.(0.005)]$

$= 3\cos\left(\frac{\pi}{2}n\right)$ discrete-time signal obtained after sampling at 200Hz.

c) $F_s = 75$ Hz. $\Rightarrow$ $T_s = \frac{1}{75}$

$x[n] = x(nT_s) = 3\cos[100\pi n \frac{1}{75}] = 3\cos\left(\frac{4\pi}{3}n\right)$

$= 3\cos\left[\left(2\pi - \frac{2\pi}{3}\right)n\right] = 3\cos\left(2\pi n - \frac{2\pi}{3}n\right)$

$= 3\cos\left(\frac{2\pi}{3}n\right)$ discrete-time signal obtained after sampling at 75 Hz.

d)

let's call $y(t)$ the sinusoid that gives identical samples to those in part c)

$y(t) = 3\cos(2\pi f_0 t)$ sample with 75 Hz. $\Rightarrow T_s = \frac{1}{75}$ sec.

$y(nT_s) = 3\cos\left(2\pi f_0 n \frac{1}{75}\right)$

let's make $y(nT_s) = x(nT_s)$ (because of identical samples condition)

$\Rightarrow \cancel{3}\cos\left(2\pi f_0 \frac{n}{75}\right) = \cancel{3}\cos\left(\frac{2\pi}{3}n\right)$

$\Rightarrow \frac{\cancel{2\pi} f_0 \cancel{n}}{75} = \frac{\cancel{2\pi}}{3}\cancel{n}$ $\Rightarrow f_0 = \frac{75}{3} = 25$ Hz.

$\Rightarrow y(t) = 3\cos(2\pi(25)t) = 3\cos(50\pi t)$ gives the same (identical)

Q.7 Let $x(t) = 7\sin(11\pi t)$. In each of the following parts, the discrete-time signal $x[n]$ is obtained by sampling $x(t)$ at a sampling rate f_s; and the resultant discrete-time signal $x[n]$ is written:

$$x[n] = A\cos(2\pi \hat{f}_o n + \phi)$$

For each part below, determine the values A, ϕ and $\hat{f}_o$. In addition, state whether or not the signal has been over-sampled or under-sampled.

Definitions:

over-sampling: Sampling with a sampling frequency which is much higher than Nyquist rate

$$f_s >> \text{Nyquist rate} = 2 \cdot f_{max} \leftarrow \text{max. freq. of cts-time signal}$$

under-sampling: sampling with a sampling frequency which is less than the Nyquist rate

$$f_s < \text{Nyquist rate} = 2 \cdot f_{max}$$

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a) $f_s = 10$ Hz. (=10 samples/sec.) $\Rightarrow T_s = \frac{1}{f_s} = 0.1$ sec.

T_s: sampling interval

$\Rightarrow x[n] = x(nT_s) = 7\sin(11\pi n \cdot 0.1) = 7\sin(1.1\pi n)$

note that $f_{max} = 5.5$ Hz. (freq. of cts-time signal) $\Rightarrow$ Nyquist rate $= 11$ Hz.

$f_s = 10^{Hz} <$ Ny. rate $= 11^{Hz}$. (undersampled)

b) $f_s = 15$ Hz. $\Rightarrow T_s = \frac{1}{15}$ sec.

$\Rightarrow x[n] = x(nT_s) = 7\sin(11\pi n \frac{1}{15}) \cong 7\sin(0.73\pi n)$

this time

$f_s = 15$ Hz $>$ Ny. rate $= 11$ Hz. in

Question :

Consider the analog signal
$x_a(t) = 3\cos(2000\pi t) + 5\sin(6000\pi t) + 10\cos(12000\pi t)$.

a) What is the Nyquist rate?

Answer: $f_1 = 1000$ Hz, $f_2 = 3000$ Hz, $f_3 = 6000$ Hz.
$\Rightarrow f_{max} = 6000$ Hz.
$\Rightarrow$ Nyquist rate $= 12000$ Hz.
$f_s > 12{,}000$ Hz.

b) Assume we sample using sampling rate $F_s = 5000$ samples/sec. What is the discrete-time signal $x[n]$ obtained after sampling?

Answer: $x[n] = 3\cos\left(2000\pi \frac{n}{5000}\right) + 5\sin\left(\frac{6000\pi n}{5000}\right) + 10\cos\left(\frac{12000\pi n}{5000}\right)$

$= 3\cos\left(\frac{2\pi}{5}n\right) + 5\sin\left(\frac{6\pi}{5}n\right) + 10\cos\left(\frac{12\pi}{5}n\right)$

$= 3\cos\left(\frac{2\pi}{5}n\right) + 5\sin\left[\left(2\pi - \frac{4\pi}{5}\right)n\right] + 10\cos\left[\left(2\pi + \frac{2\pi}{5}\right)n\right]$

$= 3\cos\left(\frac{2\pi}{5}n\right) - 5\sin\left(\frac{4\pi}{5}n\right) + 10\cos\left(\frac{2\pi}{5}n\right)$

$= 13\cos\left(\frac{2\pi}{5}n\right) - 5\sin\left(\frac{4\pi}{5}n\right) = 13\cos\left[2\pi\left(\frac{1}{5}\right)n\right] - 5\sin\left[2\pi\left(\frac{2}{5}\right)n\right]$

Is there aliasing?

c) if we want to recunstruct the analog signal from $x[n]$, which signal do we get back?

Answer:
$y_a(t) = 13\cos[2\pi(1000)t] - 5\sin[2\pi(2000)t]$

which is different from the original signal

$f_1 = F_1 \cdot f_s = \frac{1}{5} \cdot 5000 = 1000$ H
$f_2 = F_2 \cdot f_s = \frac{2}{5} \cdot 5000 = 2000$ H

Question:

Suppose that a discrete-time signal is given by the formula $x[n] = 10\cos(0.2\pi n - \frac{\pi}{7})$ and that it was obtained by sampling a continuous time signal at a sampling rate of $f_s = 1000$ samples/sec. Determine two <u>different</u> continuous-time signals $x_1(t)$ and $x_2(t)$ whose samples are equal to $x[n]$. That is; find $x_1(t)$ & $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 0.001$ sec. Both of thes continuous-time should have frequencies less than 1000 Hz. What are f_1 and f_2? Write down formulas of $x_1(t)$ and $x_2(t)$.

Answer:

$x[n] = 10\cos(0.2\pi n - \frac{\pi}{7})$ $\quad f_s = 1000$ samples/sec.

let $\underline{x_1(t) = 10\cos(2\pi f_1 t - \frac{\pi}{7})}$

$$x[n] = x_1\left(\frac{n}{1000}\right) = 10\cos\left(\frac{2\pi f_1 n}{1000} - \frac{\pi}{7}\right) \Rightarrow 0.2\pi = \frac{2\pi f_1}{1000}$$

$\Rightarrow \underline{f_1 = 100 \text{ Hz}}.$

$$x[n] = 10\cos\left(\frac{2\pi n}{10} - \frac{\pi}{7}\right) = 10\cos\left[2\pi n - \left(\frac{2\pi n}{10} - \frac{\pi}{7}\right)\right]$$

$$= 10\cos\left(2\pi n - \frac{2\pi n}{10} + \frac{\pi}{7}\right) = 10\cos\left(2\pi \frac{9}{10} n + \frac{\pi}{7}\right)$$

$$\Rightarrow \underline{x_2(t) = 10\cos\left(2\pi f_2 t + \frac{\pi}{7}\right)}$$

$$\Downarrow$$

$$f_2 = F_2 \cdot f_s = \frac{9}{10} \cdot 1000 = \underline{900 \text{ Hz}}.$$

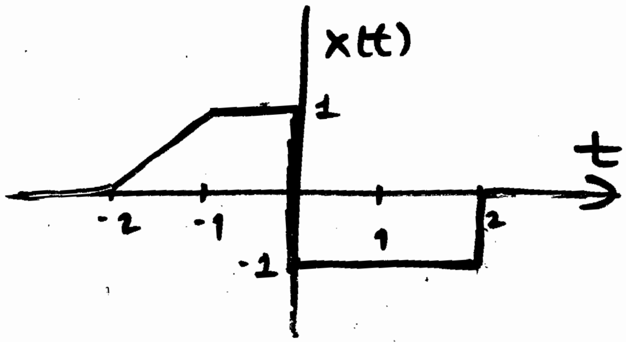
Check:

$$x[n] = 10\cos\left(2\pi \frac{900}{1000} n + \frac{\pi}{7}\right)$$

$$= 10\cos\left(2\pi (0.9) n + \frac{\pi}{7}\right) = 10\cos\left(1.8\pi n + \frac{\pi}{7}\right)$$

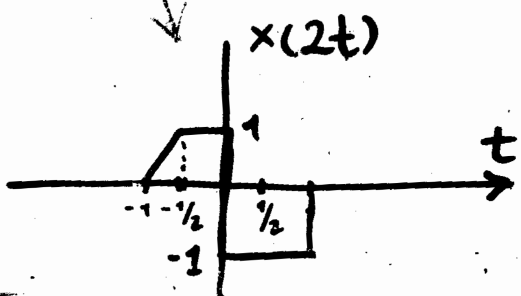
$$= 10\cos\left[2\pi n - \left(1.8\pi n + \frac{\pi}{7}\right)\right] = 10\cos\left(0.2\pi n - \frac{\pi}{7}\right)$$

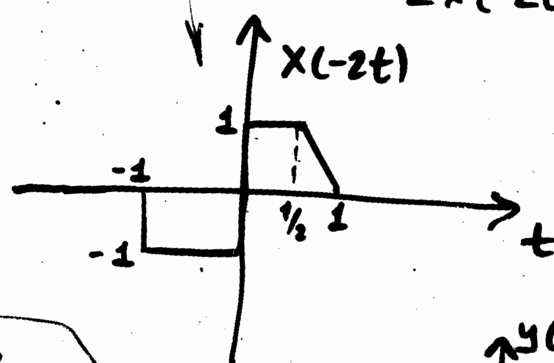
Question: $x(t)$ is drawn in the following figure. Sketch and label $y(t) = x(1-2t)$.

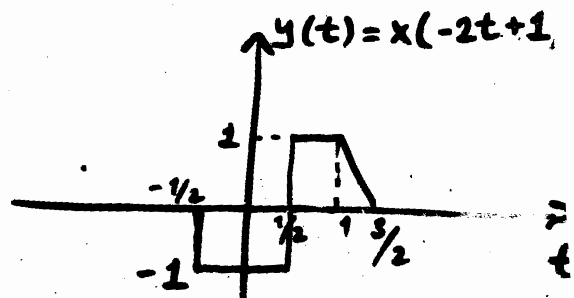


$$x(t) \xrightarrow[\text{by 2}]{\text{compress}} x(2t) \xrightarrow{\text{axis reversal}} x(-2t) \xrightarrow{\text{delay by } 1/2} x[-2(t-1/2)]$$

$$= x(-2t+1) = y(t)$$

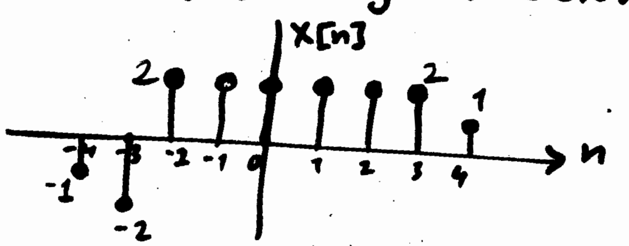






Check the answer!

Question: $x[n]$ is given below.



Sketch $y[n] = x[2-n]$.

$$x[n] \xrightarrow{\text{axis reversal}} x[-n] \xrightarrow{\text{delay by 2}} x[-(n-2)] = x[2-n] = y[n]$$