# Dokuz Eylül University <br> Department of Electrical and Electronics Engineering 

EE 5150-Transform Theory and Its Applications<br>Final Exam, Fall 2009

January 18th, 2010; 10:00 AM-17:00 PM

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## Student No:

Student Name:

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
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Q1) Consider a discrete time system described by the linear shift invariant difference equation of second order given below.

$$
y(n)=0.3 y(n-1)-0.02 y(n-2)+x(n)-0.1 x(n-1)
$$

where $x(n)=(-0.2)^{2} u(n), y(-1)=1.0$ and $y(-2)=0.6$. Find the output response of the system in time domain.

Q2) The correlation matrix of a vector is given by

$$
\mathrm{R}_{\mathrm{x}}=\left[\begin{array}{ccc}
0.3 & 0.1 & 0.1 \\
0.1 & 0.3 & -0.1 \\
0.1 & -0.1 & 0.3
\end{array}\right]
$$

Calculate the Karhunen-Loewe transform of the input vector.

Q3) Find the Hilbert transform of $\mathrm{g}(\mathrm{t})=1 / \mathrm{t}$.

Q4) If $\emptyset_{1}(\mathrm{t})$ and $\emptyset_{2}(\mathrm{t})$ satisfy dilation equation,
(a) Does their product $\mathrm{P}(\mathrm{t})=\emptyset_{1}(\mathrm{t}) \emptyset_{2}(\mathrm{t})$ satisfy a dilation equation? Justify your answer.
(b) Does their convolution $\mathrm{P}(\mathrm{t})=\emptyset_{1}(\mathrm{t}) * \emptyset_{2}(\mathrm{t})$ satisfy a dilation equation? Justify your answer.

Q5) Consider $\mathrm{h}(\mathrm{n})=\left(\frac{2}{5 \sqrt{2}}, \frac{6}{5 \sqrt{2}}, \frac{3}{5 \sqrt{2}}, \frac{-1}{5 \sqrt{2}}\right)$. Find the corresponding wavelet filter for an orthogonal wavelet system.

Q6) The discrete cosine transform of type III is given by

$$
\operatorname{DCT}_{\mathrm{N}}^{\mathrm{III}}=\frac{\sqrt{2}}{\mathrm{~N}} \sum_{\mathrm{k}} \mathrm{a}_{\mathrm{k}} \cos \left[\frac{(2 \mathrm{n}+1) \mathrm{k} \pi}{2 \mathrm{~N}}\right] ; \quad \mathrm{k}, \mathrm{n}=0,1, \cdots, \mathrm{~N}
$$

Find the DCT matrix for $\mathrm{N}=6$, and skecth it.

Q7) Find the Laplace transform of

$$
\frac{\mathrm{d}^{2} \mathrm{x}}{\mathrm{dt}^{2}}+2 \frac{\mathrm{dx}}{\mathrm{dt}}-3 \mathrm{x}=\mathrm{e}^{\mathrm{t}}
$$

where $x(0)=1$ and $\frac{\mathrm{dx}(0)}{\mathrm{dt}}=-1$.

Q8) Find the inverse Laplace transform of $\frac{s}{\left(s^{2}+\omega^{2}\right)^{2}}$ using convolution property of the Laplace transform.

Q9) Compute the inverse Fourier transform of

$$
F(\omega)=\left(1+\omega^{2}\right)^{-1}
$$

Q10) Compute the N -point DFTs of the following signal.

$$
x(n)=\cos \left(\frac{2 \pi}{N} k_{0} n\right), \quad 0 \leq n \leq N-1
$$

