Dokuz Eylül University

Department of Electrical and Electronics Engineering

EE 5150–Transform Theory and Its Applications

Final Exam, Fall 2009

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S1) $\mathrm{Y(z) = 0.3[z^{-1}Y(z) + 0.3y(-1)] - 0.02[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)] + X(z) - 0.1[z^{-1}X(z) + x(-1)]}$

We know that $\mathrm{X(z) = z/(z+0.2)}$ and x(-1)=0. Then

$$Y(z) = \frac{0.3y(-1) - 0.02\mathrm{z}^{-1}\mathrm{y}(-1) - 0.02\mathrm{y}(-2)}{1 - 0.3\mathrm{z}^{-1} + 0.02\mathrm{z}^{-2}} + \frac{(1 - 0.1\mathrm{z}^{-1})X(z)}{1 - 0.3\mathrm{z}^{-1} + 0.02\mathrm{z}^{-2}}$$

When the input x(n) is zero, X(z)=0 then the second term on the right side is zero. It is the z transform of the zero input response $y_{0i}(n)$. The inverse z transform of this first term on the right side.

$$\mathrm{Y_{0i}(z) = \frac{0.3y(-1) - 0.02z^{-1}y(-1) - 0.02y(-2)}{1 - 0.3z^{-1} + 0.02z^{-2}} = \frac{0.288 - 0.02z^{-1}}{1 - 0.3z^{-1} + 0.02z^{-2}}}$$
$$\mathrm{= \frac{0.288z^2 - 0.02z}{z^2 - 0.3z + 0.02}}$$

$$\frac{\mathrm{Y_{0i}(z)}}{z} = \mathrm{\frac{z(0.288z - 0.02)}{(z-0.1)(z-0.2)} = \frac{0.376z}{(z-0.2)} - \frac{0.088}{(z-0.1)}}$$

$$\mathrm{y_{0i}(n) = [0.376(0.2)^n - 0.088(0.1)^n]u(n)} \quad \rightarrow \quad \text{zero input response}$$

$$\mathrm{Y_{0s}(z) = \frac{X(z)(1 - 0.1z^{-1})}{1 - 0.3z^{-1} + 0.02z^{-2}} = \left[\frac{1 - 0.1z^{-1}}{1 - 0.3z^{-1} + 0.02z^{-2}}\right]\frac{z}{z+0.2}}$$
$$\mathrm{= \frac{z^2(z-0.1)}{(z+0.2)(z-0.1)(z-0.2)} = \frac{z^2}{(z+0.2)(z-0.2)} = \frac{-0.5z}{(z+0.2)} + \frac{0.5z}{(z-0.2)}}$$

$$\mathrm{y_{0s}(n) = 0.5[-(-0.2)^n + (0.2)^n]u(n)} \quad \rightarrow \quad \text{zero state response}$$

S2) For this case we have

$$a_0 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\-1\\-1\end{bmatrix}, a_1 = \frac{1}{\sqrt{6}}\begin{bmatrix}2\\1\\1\end{bmatrix}, a_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

The Karhunen-Loewe transform is then given by

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \end{bmatrix}$$

S3) $H[\delta(t)] = \frac{1}{\pi t}$ and $H\left[\frac{1}{\pi t}\right] = -\delta(t)$. Then $H\left[\frac{1}{t}\right] = -\pi\delta(t)$.

S4) If $\emptyset_1(t)$ and $\emptyset_2(t)$ satisfy dilation equation,

(a) Does not satisfy.

(b) Will satisfy a dilation equation as follows.

$$\Phi_1(\omega) = \frac{1}{\sqrt{2}} H^1\left(\frac{\omega}{2}\right) \Phi_1\left(\frac{\omega}{2}\right)$$

$$\Phi_2(\omega) = \frac{1}{\sqrt{2}} H^2\left(\frac{\omega}{2}\right) \Phi_2\left(\frac{\omega}{2}\right)$$

$$P(\omega) = \Phi_1(\omega).\Phi_2(\omega) = \frac{1}{2} H^1\left(\frac{\omega}{2}\right) H^2\left(\frac{\omega}{2}\right) \Phi_1\left(\frac{\omega}{2}\right) \Phi_2\left(\frac{\omega}{2}\right) = \frac{1}{\sqrt{2}} H\left(\frac{\omega}{2}\right) P\left(\frac{\omega}{2}\right)$$

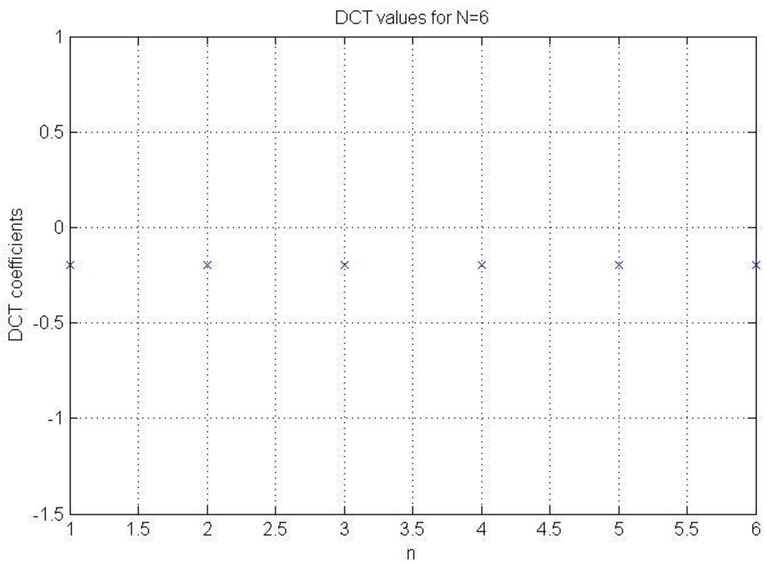
where $H(\omega) = \frac{1}{\sqrt{2}} H^1(\omega) H^2(\omega)$.

S5) $h_1(n) = -1^n h(N-n)$. Therefore,

$$h_1(0) = h(3) = \frac{-1}{5\sqrt{2}}, h_1(1) = -h(2) = \frac{-3}{5\sqrt{2}}, h_1(2) = h(1) = \frac{6}{5\sqrt{2}}, h_1(3) = -h(0) = \frac{-2}{5\sqrt{2}}$$

S6) $DCT_6^{III}(k) = \frac{\sqrt{2}}{6}\left[\frac{1}{12} + \frac{cos\,(2k+1)\pi}{12} + \frac{cos\,(2k+1)\pi}{6} + \frac{cos\,(2k+1)\pi}{4} + \frac{cos\,(2k+1)\pi}{3} + \frac{cos\,(2k+1)5\pi}{12}\right]$

$$DCT_6^{III}(0) = DCT_6^{III}(1) = DCT_6^{III}(2) = DCT_6^{III}(3) = DCT_6^{III}(4) = DCT_6^{III}(5) = -0.197$$



S7) $L\left\{\frac{d^2x}{dt^2}+2\frac{dx}{dt}-3x\right\}=L\{e^t\}$

$$s^2X(s)-sx(0)-\frac{dx(0)}{dt}+2[sX(s)-x(0)]-3X(s)=\frac{1}{s-1}$$

$$s^2X(s)-1\mp 1+2sX(s)-2-3X(s)=\frac{1}{s-1}$$

$$(s^2+2s-3)X(s)=\frac{1}{s-1}+2=\frac{2s-1}{s-1}$$

$$X(s)=\frac{2s-1}{(s-1)(s^2+2s-3)}$$

S8) $L^{-1}\left\{\frac{s}{(s^2+\omega^2)^2}\right\}=L^{-1}\left\{\frac{s}{s^2+\omega^2}\cdot\frac{s}{s^2+\omega^2}\right\}=L^{-1}\left\{\frac{s}{s^2+\omega^2}\right\}*L^{-1}\left\{\frac{s}{s^2+\omega^2}\right\}=\cos(\omega t)*\frac{\sin(\omega t)}{\omega}=$
$\frac{1}{\omega}\int_0^t\cos(\omega x)\sin[\omega(t-x)]dx=\frac{1}{\omega^2}\left[\frac{1}{4}\cos(-2\omega x+\omega t)+\frac{1}{2}(t\omega)\sin(\omega t)\right]\Big|_0^t=$
$\frac{1}{\omega^2}\left[\frac{1}{4}\cos(\omega t)+\frac{1}{2}(t\omega)\sin(\omega t)\right]-\frac{1}{\omega}\left[\frac{1}{4}\cos(\omega t)\right]=\frac{t}{2\omega}\sin(\omega t)$

Q9) Compute the inverse Fourier transform of

$$f(t)=F^{-1}\{F(\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{j\omega t}}{1+\omega^2}d\omega$$

for t=0, $f(0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{1+\omega^2}d\omega=\frac{1}{2\pi}\arctan(\omega)|_{-\infty}^{+\infty}=\frac{1}{2}$

to evalute f(t) when $t\neq 0$,

$$f(t)=\frac{1}{2\pi}\left[\pi e^{-t}-\lim_{k\to\infty}\int\frac{e^{jtz}}{1+z^2}dz\right]=\frac{1}{2}e^{-t}$$

Q10) $x(n)=\frac{1}{2}e^{j\frac{2\pi n}{N}k_0}+\frac{1}{2}e^{-j\frac{2\pi n}{N}k_0}=x_1(n)+x_2(n)$

$$X_1(k)=\sum_{n=0}^{N-1}e^{j\frac{2\pi n}{N}k_0}e^{-j\frac{2k\pi n}{N}}=\sum_{n=0}^{N-1}e^{j\frac{-2\pi(k-k_0)n}{N}}$$

From this result

$$X(k)=\frac{N}{2}[\delta(k-k_0)+\delta(k-N+k_0)]$$