Dokuz Eylül Üniversity

Department of Electrical and Electronics Engineering

EE 5150–Transform Theory and Its Applications

Midterm Exam, Fall 2009

November 23th, 2009, 9:30-12:15 AM

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Student No:
Student Name:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13

TOTAL:	

Q1) Determine the impulse response **(3p.)** and the step response of the following causal system **(3p.)**. Are the system stable? **(3p.)**

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$$

Q13) Find the Fourier sine transform of $e^{-x}, \;\; x \geq 0$ **(2p.)**. By using this result, show that **(3p.)**

$$\int_0^\infty \frac{x\sin(mx)}{x^2+1}dx = \frac{\pi}{2}e^{-m}, \qquad m > 0$$

Q2) If X(z) is the z transform of x(n), show that **(8p.)**

$$Z\{x^*(n)\} = X^*(z^*)$$

Q3) Determine thr causal signal x(n) if its z transform, X(z), is given by **(8p.)**

$$X(z) = \frac{1 + 3z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$$

Q12) Laplace transforms are useful for solving linear differential equations that arise in many settings. Consider a linearized model of a motor with a torque control system, for example, where the actual torque of the motor relative to a desired torque is described by the following Laplace transform.

$$H(s) = \frac{\frac{k_p}{L}\left(s + \frac{k_I}{k_p}\right)}{s^2 + \frac{k_p}{L}s + \frac{k_I}{L}}$$

where L=1 mH=inductance of motor windings, k_p=gain for proportional feedback, k_I=gain for integral feedback. The inverse Laplace transform of H(s) contains multiplicative factors of the form e^{-at} that determine how fast time waveforms decay to zero. This decay corresponds to how rapidly the motor torque reaches a desired value. A faster decay is desirable. Equal roots, (i.e. critical damping), is optimal if vibration (oscillatory solutions) must be eliminated. Find the inverse Laplace transform of H(s) for the values; a=1.6k, k_I=2.56G. **(10p.)**

Q11) Find the Laplace transform of f(t). **(6p.)**

$$f(t) = e^{-2t}\cos(\sqrt{3}) - t^2 e^{-2t}$$

Q4) Let x(n) be a sequence with z transform X(z). Determine, in terms of X(z), the z transform of the following signal. **(8p.)**

$$x_1(n) = x(2n)$$

Q5) Determine the signal x(n) if its Fourier transform is a given as **(6p.)**

Q10) Consider a stationary source with variance of unity and normalized autocorrelation ρ between neighboring samples

$$R_{xx} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

Find the eigenvalues, eigenvectors and the coefficients of KLT. **(10p.)**

Q9) The DCT is defined as

$$\theta(k) = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} \alpha(k)x(n)\cos\left[\frac{(2n+1)k\pi}{2N}\right] \qquad k=0, 1, \cdots, N-1$$

$$x(n) = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} \alpha(k)\theta(k)\cos\left[\frac{(2n+1)k\pi}{2N}\right] \qquad k=0, 1, \cdots, N-1$$

where $\alpha(0) = \frac{1}{\sqrt{2}}$ and $\alpha(k) = 1; \quad k \neq 0.$

For N=2, calculate the DCT **(2p.)**. What is the identical transform of this result included? **(4p.)**

Q6) Consider a signal $x(n)=\{-1, 2, \underset{\uparrow}{3}, 2, -1\}$ with Fourier transform $X(\omega)$. Compute the following quantities, without explicity computing $X(\omega)$;

(a) $X(0)$, **(2p.)** (b) Angle$\{X(\omega)\}$, **(2p.)** (c) $X(\pi)$, **(2p.)** (d) $\int_{-\pi}^{\pi}|X(\omega)|^2 d\omega$, **(2p.)** (e) $\int_{-\pi}^{\pi} X(\omega) d\omega$ **(2p.)**

Q7) Let $x_a(t)$ be an analog signal with bandwith B=3 kHz. We wish to use $N=L^m$-point DFT to compute the spectrum of the signal with a resolution less than or equal to 50 Hz. Determine the minimum sampling rate **(2p.)**, the minimum number of required samples **(2p.)** and the minimum length of the analog signal record **(2p.)**.

Q8) Compute the N-point DFTs of the following signal. **(8p.)**

$$x(n) = \cos\left(\frac{2\pi}{N} k_0 n\right), \qquad 0 \le n \le N-1$$